Scam the Declined or Decline to be Scammed: A Model of Financial Choice in the Presence of Cognitive Decline*

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Abstract

We develop a multi-period overlapping generations model in which cognitively sound consumers know that their future selves may suffer from cognitive decline, but underestimate their likelihood of cognitive decline. Consumers with cognitive decline are at risk being taken advantage of by fraudsters who steal a portion of a consumer's current consumption through scams. Cognitively sound consumer are not at risk of being scammed. Prior to being hit by cognitive decline, consumers can pay to preemptively protect their future selves from fraudsters by paying a financial advisor an ongoing fee. In practice such protection could take many forms such as hiring a fiduciary advisor who monitors and participates in a consumer's future financial decisions.

In equilibrium, more consumers pay for protection from fraudsters and fraudsters enter the market more intensely when the probability of consumers being afflicted by cognitive decline is high, life expectancy is high, average potential losses to consumers are high, and the cost to fraudsters of meeting more consumers is low. The cost of consumer protection and the intensity with which fraudsters enter the market are determined endogenously. When a higher fraction of consumers protect themselves, fraudsters enter the market with lower intensity. Thus when the probability of a consumer being offered a fraudulent product is high, consumers who purchase protection provide a small positive externality to other consumers. Since consumers underestimate their likelihood of decline, in equilibrium fewer consumers purchase protection than is socially optimal.

1 Introduction

Nearly everyone with a telephone or email address has been offered fraudulent financial opportunities. Most of us realize that no Nigerian official truly intends to share $60 million with us, that the IRS does not call threatening to arrest people who won't provide immediate credit card payment over the phone, and that high return investments are never guaranteed. However, not everyone is able to distinguish the real from the fraudulent. Even legitimate financial products, such as home equity loans or variable annuities, may present a hazard to those who are unable to determine whether products are suitable to their needs or reasonably priced.

Over 5.1 million Americans have Alzheimer's disease. By 2050, that number is expected to grow to between 11 million and 16 million (Alzheimer's Association, 2015). Approximately 24% of people between 80 and 89 years old suffer from dementia (Plassman et al., 2007) while 29% have cognitive impairment without dementia (Plassman et al., 2008). It is estimated that 75% of those over 90 have dementia or impairment. Cognitive impairment is a primary risk factor for financial abuse (Heath et al., 2005). Even older persons without dementia but with cognitive decline are more susceptible to financial scams (Boyle et al., 2012); and age-related declines in memory make older adults more susceptible to scams (Jacoby et al., 2005; Jacoby, 1999). Unfortunately, those suffering from cognitive decline usually do not recognize or acknowledge their loss of the ability to make sound financial decisions. Confidence in financial decision-making does not decrease in old age and may even increase (Finke, Howe, and Huston, 2017).

In the cross-section, "some aspects of age related cognitive decline begin in healthy educated adults when they are in their 20s and 30s" (Salthouse, 2009). While household financial decision making is strongly correlated with cognitive ability (Agarwal and Mazumder, 2013), practical knowledge, often gained through experience, is also critical for many financial tasks. Agarwal et al. (2009), propose that the negative relationship of age and cognition coupled with a positive relationship between age and experience, lead to a hump-shaped relationship between age and the ability of adults to make financial decisions. They estimate that for many household finance tasks, decision-making ability peaks, on average, at about 53 years old.

As the US population ages, more wealth is controlled by those at risk of cognitive decline. 23% of U.S. households have a head 65 or older and, in aggregate, these households own 36% of all U.S. household assets and 40% of U.S. household equity in homes (U.S. Census Bureau, 2014). Home equity lines of credit and refinancing make it increasingly easy for consumers to access the wealth in their homes, while the shift from defined benefit to defined contribution pensions gives retirees direct access to their retirement savings. Meanwhile changes in technology have made it increasingly easy for scammers to make contact with the elderly and to defraud them in novel ways (Blanton, 2012; Deem, Nerenberg, and Titus, 2007).

In this article, we develop a a multi-period overlapping generations model in which consumers decide whether or not to protect their future old selves from bad financial decisions due to cognitive decline. Consumers are born with good cognitive health but do not know whether or when they will suffer from decline and underestimate the true probability of cognitive decline. Consumers with cognitive decline are at risk being taken advantage of by fraudsters. Consumers die stochastically and are replaced by young consumers. Fraudsters pay to increase the probability of meeting consumers but are unable to identify ex-ante the cognitive state of each consumer or whether a consumer has purchased protection. Prior to being hit by cognitive decline, consumers can preemptively protect their future selves from fraudsters by paying a financial advisor an ongoing fee. In practice such protection could take many forms such as hiring a fiduciary advisor who monitors and participates in a consumer's future financial decisions.

In equilibrium, more consumers protect against bad financial decisions due to cognitive decline when the probability of being afflicted by cognitive decline is initially high, life expectancy is high, average potential losses are high, and the cost to fraudsters of meeting more consumers is low. Fraudsters are more active in the market when the cost of meeting more consumers is low, the probability of consumers being afflicted by cognitive decline is high, life expectancy is high, and the average potential gains from defrauding consumers are high. The cost of consumer protection and the intensity with which fraudsters enter the market are determined endogenously.

In our model, when a higher fraction of consumers protect themselves, fraudsters enter the market with lower intensity. Thus when the probability of a consumer being offered a fraudulent product is high, consumers who purchase protection provide a positive externality to other consumers. Since consumers underestimate their probability of cognitive decline, in equilibrium fewer consumers purchase protection than is socially optimal. In other words, when fraud and cognitive decline are prevalent, a social planner would choose a higher equilibrium rate of protection than consumers choose.

Agents with time inconsistent preferences may anticipate that their future selves will engage in more short-term consumption that the agent believes to be optimal in the long run. Strotz (1956) introduced the use of pre-commitment to deal with time-inconsistent preferences. Laibson (1997) developed a commitment technology in which a person with hyperbolic discounting purchases illiquid assets so as to prevent his future, impatient, self from consuming more that his current self considers optimal. Time inconsistent preferences can also lead to procrastination which an agent realizes not to be in his own best long term interests. Bond and Sigurdsson (2018) demonstrate that even with random shocks to wealth or time commitments, contracts can be used to influence the consumption and procrastination behavior of future selves by taking advantage of time inconsistency driven preference reversals.

In our model, we introduce pre-commitment to deal with a different form of intertemporal inconsistency. In our model, consumers' preferences are time consistent. However, consumers realize that their future selves may not have the cognitive ability to make sound financial decisions and may fall prey to fraudsters. Thus, the future self may make bad decisions that the current self would not make. Rather than risk their future selves inflicting self-harm through bad purchase decisions, consumers restrict their own future behavior.

Our model is related to prior work on crime and predators. Cook's (1986) criminal opportunity theory postulates that criminal activity will increase with opportunity and that potential victims will respond to the threat of crime by engaging in self-protection efforts. In our model, fraudsters engage more intensely with consumers when dementia is high, levels of protection low, and potential gains high, that is, opportunities are plentiful. Consumers respond to the threat of scams through self-protection. Erlich (1996) decomposes the expected net return to a crime into the direct costs of engaging in crime minus the foregone wages minus the probability of convictions times the prospective penalty if convicted. While fraudsters in our model pay a cost to engage in crime (which could include opportunity costs), we do not model the prospect of being caught, convicted, and punished. Doing so would, of course, reduce the intensity of scams and the demand for protection.

Conlisk (2001) develops a model in one example of which people choose to be "tricksters," "avoiders," or "suckers." It is costlier to become a trickster than an avoider and costlier to become an avoider than a sucker. Each period people are randomly paired and tricksters take money from suckers but not from other tricksters or avoiders. Avoiders neither take money from nor give money to anyone, i.e., are neither perpetrators or victims of scams. Suckers are exploited by tricksters but not by avoiders or other suckers. All three types can co-exist in equilibrium. In our model, fraudsters prey on unprotected consumers who have cognitive decline but are not protected. Consumers in our model change types with the unintentional onset of cognitive decline and with the intentional purchase of protection.

2 The Model

2.1 Setup and Assumptions

We consider a discrete-time infinite-horizon model in which each of a mass one of *consumers* face the threat of their consumption being lowered through scams. We refer to perpetrators of scams as *fraudsters.*

Consumers are risk-neutral and all born with the wealth $W_0 > 0$, which generates a consumption stream $c = rW_t$ each period until they die, where $r > 0$ is the exogenous one period interest rate.

In every period, each consumer (of any age) faces a constant probability $q \in (0, 1)$ of dying, and all deceased consumers are replaced with the same mass of newborn consumers aged zero.

In the period in which they are born (e.g., retire), consumers are not yet affected by cognitive decline. We refer to them as *competent* consumers (CC). Competent consumers can identify fraudsters and are not vulnerable to being scammed.

Competent consumers decide whether or not to purchase, from an advisor, scam protection for the next period. Protection has a per period cost of p, a quantity to be endogenized later. Because the decision facing a competent consumer does not change from period to period, each competent consumer will make the same decision about purchasing protection in each period.

Competent consumers die each period with probability q. If they do not die, they either enter *cognitive decline* (CD) with probability δ or they remain competent with probability $(1-\delta)$.

If a consumer who enters cognitive decline has previously purchased protection, he continues to purchase protection for the rest of his life; if he has not previously purchased protection he never purchases it.

The objective probability of a competent consumer entering cognitive decline in the next period is δ. Once in a state of cognitive decline, a consumer remains there until he dies.

Competent consumers inaccurately believe that their probability of entering cognitive decline is smaller than it actually is. Each consumer i believes that his probability of entering decline is $\zeta_i\delta$, $\zeta_i \overset{\text{iid}}{\sim} \text{U}[0,1]$.

In each period, every living consumer encounters a fraudster with probability $\phi_t \in [0,1]$, a quantity to be endogenized later. ϕ_t is the same for every consumer because fraudsters cannot target consumers based on their age, cognitive state, or purchase of protection When a consumer encounters a fraudster, his ability to turn down the avoid being scammed depends on whether or not the consumer suffers from cognitive decline and whether or not he has purchased protection at a cost of p.

If targeted by a fraudster and unprotected, the consumer with cognitive decline loses one period consumption c. If protected and targeted by a fraudster, the consumer pays p for protection but does not lose additional consumption.

Fraudsters choose the intensity with which they target consumers in every period. Specifically, we assume that fraudsters can increase the probability with which they meet each consumer to ϕ_t at a cost of $c(\phi_t)$, where $c(0) = 0$, $c'(\phi) > 0$, $c''(\phi) \geq 0$, and $c(1)$ is sufficiently large to ensure that the optimal ϕ_t is always strictly smaller than one. For simplicity, we initially assume that $c(\phi) = \frac{k}{2}\phi$, with k sufficiently large to ensure interior solutions. In their choice of ϕ_t, fraudsters seek to maximize the total (expected) amount they can appropriate from consumers through fraud, net of search costs.

When a fraudster targets a competent consumer, the fraudster gains nothing. When a fraudster targets an unprotected consumer with cognitive decline, the fraudster gains c with probability 1 and the consumer loses c.

Advisors are competitive and set the per period fee p to be charged to consumers for protecting their consumption c to the advisors' exogenously determined cost of providing protection, ρ. This can be thought of as the cost of monitoring protected consumers' purchases.

2.2 Steady State

Throughout the paper, we are interested in characterizing the steady-state equilibrium, in which the consumer population's age profile remains the same. As such, from here on, we use $t \in \{0, 1, \ldots\}$ to denote the consumer population of age t. Given the unit mass of consumers, the constant probability of dying and automatic replacement of those who die, and the assumption that one's experience in the financial market does not affect one's life expectancy, the steady-state economy has a mass $n_0 = q$ of consumers aged zero, $n_1 = q(1-q)$ consumers aged one and, more generally, a mass $n_t = q(1-q)^t$ of consumers aged $t \in \{0, 1, \ldots\}$. Because all consumers are born with the same wealth and consumption, it is also the case that, in the steady-state equilibrium, the search intensity of fraudsters is constant at ϕ, and that the price of insurance is set at a constant p by insurers.

To summarize, the sequence of events for any consumer i born with consumption c_0 is as follows in each period. Every period starts with the insurers setting the publicly available fee p of insurance and with fraudsters setting the unobservable search intensity ϕ with which they reach consumers. A competent consumer i enters period t believing that his probability of entering cognitive in the next period is $\zeta\delta$. Based on $\zeta\delta$ and p, the consumer decides whether or not to purchase protection from the advisor. This decision comes with a transfer of p each period from consumer i to the advisor. Then consumer i suffers a cognitive decline shock with probability δ and remains competent with probability $(1-\delta)$. In decline, and unprotected consumer i becomes susceptible to having his consumption, c, defrauded in this and in every subsequent period thereafter, until he dies. If protected, the consumer in decline pays p to the advisor each period but is not defrauded. If consumer i remains competent and survives the period, then he enters period $t+1$, and a similar sequence of events is repeated.

3 Equilibrium Analysis

3.1 Objective Cognitive States

Competent consumers die and are replaced by new competent consumers with probability q; competent consumers do not die and do not become compromised and thus remain competent consumers with probability $(1-q)(1-\delta)$; competent consumers enter cognitive decline with probability δ; consumers with cognitive decline die and are replaced with new competent consumers with probability q; and consumers with cognitive decline remain consumers with cognitive decline with probability $1-q$). Thus the transition probabilities between the states of Competent Consumers and Cognitive Decline are:

The steady-state probabilities are:

$$\pi_{CC} = \frac{q}{q + d(1-q)}, \quad \pi_{CD} = \frac{d(1-q)}{q + d(1-q)}$$

where π_{CC} is the true steady state fraction of an average consumers' life spent in a competent state (CC) and π_{CD} is the steady state fraction of an average consumers' life spent in a compromised state (CD). These are known to the fraudsters and advisors.

Proof:

To find the steady-state expected fraction of time in each state, we compute the stationary distribution of the Markov chain. Let the steady-state probabilities be denoted as π_{CC} and π_{CD}, where:

- π_{CC} is the steady-state probability of being in state CC.
- π_{CD} is the steady-state probability of being in state CD.

The transition matrix is:

$$P = \begin{bmatrix} P(CC \to CC) & P(CC \to CD) \\ P(CD \to CC) & P(CD \to CD) \end{bmatrix} = \begin{bmatrix} q + (1-q)(1-d) & (1-q)d \\ q & 1-q \end{bmatrix}.$$

Steady-State Equations

In the steady state, the probabilities satisfy:

$$\pi_{CC} + \pi_{CD} = 1 \quad \text{(the total probability must sum to 1)}, \tag{1}$$

$$\pi_{CC} = \pi_{CC} P(CC \to CC) + \pi_{CD} P(CD \to CC), \tag{2}$$

$$\pi_{CD} = \pi_{CC} P(CC \to CD) + \pi_{CD} P(CD \to CD). \tag{3}$$

Substitute the transition probabilities:

$$\pi_{CC} = \pi_{CC} \left(q + (1-q)(1-d)\right) + \pi_{CD} q, \tag{4}$$
$$\pi_{CD} = \pi_{CC}(1-q)d + \pi_{CD}(1-q). \tag{5}$$

Solve for π_{CC} and π_{CD}

From the first equation:

$$\pi_{CC} = \pi_{CC} \left(q + (1-q)(1-d)\right) + \pi_{CD} q.$$

Rearrange to group π_{CC} and π_{CD}:

$$\pi_{CC} \left[1 - q - (1-q)(1-d)\right] = \pi_{CD} q.$$

Simplify the term in brackets:

$$1 - q - (1-q)(1-d) = d(1-q).$$

Thus:

$$\pi_{CC} d(1-q) = \pi_{CD} q.$$

From this, express π_{CD} in terms of π_{CC}:

$$\pi_{CD} = \pi_{CC} \frac{d(1-q)}{q}.$$

Using the normalization condition $\pi_{CC} + \pi_{CD} = 1$:

$$\pi_{CC} + \pi_{CC} \frac{d(1-q)}{q} = 1.$$

Factor out π_{CC}:

$$\pi_{CC} \left(1 + \frac{d(1-q)}{q}\right) = 1.$$

Simplify:

$$\pi_{CC} = \frac{q}{q + d(1-q)}.$$

Using $\pi_{CD} = 1 - \pi_{CC}$:

$$\pi_{CD} = 1 - \frac{q}{q + d(1-q)} = \frac{d(1-q)}{q + d(1-q)}.$$

Final Result

The steady-state probabilities are:

$$\pi_{CC} = \frac{q}{q + d(1-q)}, \quad \pi_{CD} = \frac{d(1-q)}{q + d(1-q)}.$$

3.2 Subjective Cognitive States

From his own perspective competent consumer i can, each period:

- Die with probability q, or
- Survive and remain in state CC with probability $(1-q)(1-\zeta_i\delta)$, or
- Survive and transition to state CD with probability $(1-q)\zeta_i\delta$.

Once in state CD, the consumer remains there until death, so the probability of staying in state CD in a given period is $(1-q)$, and the probability of dying is q.

Thus we have a three state Markov process with an absorbing state of death, D. Let (T_{CC}) be the expected number of periods a consumer in state CC expects to spend in state CC and T_{CD} be the expected number of periods he expects to spend in state CD. We wish to calculate $\hat{F}_{CC} = T_{CC}/(T_{CD} + T_{CD})$, which is the fraction of his life he expects to spend in state CC and $\hat{F}_{CD} = T_{CD}/(T_{CD} + T_{CD})$, which is the fraction of his life he expects to spend in state CD. Note that these expectations are based on the consumer's biased belief that $\delta = \zeta\delta$.

Three-State Markov Process

Transition Matrix

The Markov transition matrix P is given by:

$$P = \begin{bmatrix} (1-q)(1-\zeta\delta) & (1-q)\zeta\delta & q \\ 0 & 1-q & q \\ 0 & 0 & 1 \end{bmatrix}.$$

Submatrix for Transient States

The submatrix Q, corresponding to transient states CC and CD, is:

$$Q = \begin{bmatrix} (1-q)(1-\zeta\delta) & (1-q)\zeta\delta \\ 0 & 1-q \end{bmatrix}.$$

Fundamental Matrix

The fundamental matrix N is defined as:

$$N = (I - Q)^{-1},$$

where I is the identity matrix. First, compute $I - Q$:

$$I - Q = \begin{bmatrix} 1-(1-q)(1-\zeta\delta) & -(1-q)\zeta\delta \\ 0 & 1-(1-q) \end{bmatrix}.$$

Simplify the entries:

$$I - Q = \begin{bmatrix} q+(1-q)\zeta\delta & -(1-q)\zeta\delta \\ 0 & q \end{bmatrix}.$$

The inverse of $I - Q$ is:

$$N = \begin{bmatrix} \frac{1}{q+(1-q)\zeta\delta} & \frac{(1-q)\zeta\delta}{q\cdot(q+(1-q)\zeta\delta)} \\ 0 & \frac{1}{q} \end{bmatrix}.$$

Expected Times in Transient States

The expected times spent in CC and CD starting from CC are given by the first row of N:

$$T_{CC} = \frac{1}{q+(1-q)\zeta\delta}, \quad T_{CD} = \frac{(1-q)\zeta\delta}{q\cdot(q+(1-q)\zeta\delta)}.$$

Ratio $\frac{T_{CC}}{T_{CC}+T_{CD}}$

The total time spent in CC and CD is:

$$T_{CC} + T_{CD} = \frac{1}{q+(1-q)\zeta\delta} + \frac{(1-q)\zeta\delta}{q\cdot(q+(1-q)\zeta\delta)}.$$

Factor out $\frac{1}{q+(1-q)\zeta\delta}$:

$$T_{CC}+T_{CD}=\frac{1}{q+(1-q)\zeta\delta}\left(1+\frac{(1-q)\zeta\delta}{q}\right).$$

The ratio $\frac{T_{CC}}{T_{CC}+T_{CD}}$ is:

$$\frac{T_{CC}}{T_{CC}+T_{CD}}=\frac{\frac{1}{q+(1-q)\zeta\delta}}{\frac{1}{q+(1-q)\zeta\delta}\left(1+\frac{(1-q)\zeta\delta}{q}\right)}.$$

Simplify:

$$\frac{T_{CC}}{T_{CC}+T_{CD}}=\frac{1}{1+\frac{(1-q)\zeta\delta}{q}}.$$

Combine terms in the denominator:

$$\frac{T_{CC}}{T_{CC}+T_{CD}}=\frac{q}{q+(1-q)\zeta\delta}.$$

Final Results

The results are:

$$T_{CC}=\frac{1}{q+(1-q)\zeta\delta},\quad T_{CD}=\frac{(1-q)\zeta\delta}{q\cdot(q+(1-q)\zeta\delta)},$$
$$\hat{F}_{CC}=\frac{T_{CC}}{T_{CC}+T_{CD}}=\frac{q}{q+(1-q)\zeta\delta}.$$

and

$$\hat{F}_{CD}=\frac{T_{CD}}{T_{CC}+T_{CD}}=\frac{(1-q)\zeta\delta}{q+(1-q)\zeta\delta}.$$

3.3 Conjectures

Advisors publicly set price, p. (Later we can attempt to endogenize ρ and p as functions of advisors conjecture about the about fraudsters' search intensity, ϕ.) For any p, there is an equilibrium that prevails between consumers and fraudsters: consumer conjecture fraudsters' search intensity and, based on that conjecture, consumers determine their optimal protection policy; similarly, fraudsters conjecture the fraction of consumers who will buy protection and based on that conjecture, fraudsters determine their optimal search intensity. In equilibrium, the conjectured quantity by each party is equal to the actual choice by the other party.

3.4 The Consumers' Problem

Suppose that consumers observe $p > 0$ and conjecture the fraudsters' search intensity to be some $\phi \in (0, 1)$. As long as a consumer i remains competent and his subjective probability of a cognitive shock entering next period remains the same (i.e., $\zeta_i\delta$), his decision to insure is the same in periods $t-1$ and t. And consumers can only consider the protection if they are still competent, as they can then anticipate the possibility that they will eventually be compromised and potentially defrauded.

Let $\hat{\zeta}$, be the ζ_i for which a consumer is indifferent between buying protection and not doing so. If the consumer does purchase protection he receives $c - p$ every period regardless of cognitive state. If he does not purchase protection, he receives c while in state CC, c while in state CD if not targeted by fraudsters, and 0 if in state CD and targeted by fraudsters. Thus, the consumer will be indifferent between buying protection and not being protected when:

$$c - p = \hat{F}_{CC}c + \hat{F}_{CD}c(1 - \phi),$$

where:

$$\hat{F}_{CC} = \frac{q}{q + (1-q)\zeta\delta}, \quad \hat{F}_{CD} = \frac{(1-q)\zeta\delta}{q + (1-q)\zeta\delta}.$$

Solution for ζ

Substitute $\hat{F}_{CC}$ and $\hat{F}_{CD}$ into the equation:

$$c - p = c\left[\frac{q}{q + (1-q)\zeta\delta} + \frac{(1-q)\zeta\delta}{q + (1-q)\zeta\delta}(1 - \phi)\right].$$

Factor out c:

$$c - p = c\left[\frac{q + (1-q)\zeta\delta(1-\phi)}{q + (1-q)\zeta\delta}\right].$$

Divide through by c (since $c > 0$):

$$1 - \frac{p}{c} = \frac{q + (1-q)\zeta\delta(1-\phi)}{q + (1-q)\zeta\delta}.$$

Multiply through by the denominator:

$$\left(1 - \frac{p}{c}\right)[q + (1-q)\zeta\delta] = q + (1-q)\zeta\delta(1-\phi).$$

Simplify:

$$\left(1 - \frac{p}{c}\right) q + \left(1 - \frac{p}{c}\right)(1-q)\zeta\delta = q + (1-q)\zeta\delta(1-\phi).$$

Group terms with ζ:

$$\zeta\left[(1-q)\delta\left(\left(1 - \frac{p}{c}\right) - (1-\phi)\right)\right] = q\frac{p}{c}.$$

Solve for ζ:

$$\zeta = \frac{q\frac{p}{c}}{(1-q)\delta\left[\left(1 - \frac{p}{c}\right) - (1-\phi)\right]}.$$

Simplification of the Solution

Multiply the numerator and denominator by c:

$$\zeta = \frac{q\frac{p}{c} \cdot c}{(1-q)\delta\left[c\left(1 - \frac{p}{c}\right) - c(1-\phi)\right]}.$$

Simplify the numerator:

$$\zeta = \frac{qp}{(1-q)\delta\left[c\left(1 - \frac{p}{c}\right) - c(1-\phi)\right]}.$$

Simplify the denominator further:

$$c\left(1 - \frac{p}{c}\right) = c - p,$$

so:

$$(c-p) - c(1-\phi) = -p + c\phi.$$

Thus:

$$\hat{\zeta} = \frac{qp}{(1-q)\delta\left(-p + c\phi\right)}.$$

Final Simplified Solution

$$\hat{\zeta} = \frac{qp}{(1-q)\delta(c\phi - p)}.$$

3.5 The Fraudsters' Problem

Fraudsters earn c when they target an upprotected consumer in cognitive decline at a cost of $\frac{k\phi}{2}$. Thus the fraudster's probem is to solve for ϕ that maximizes his after cost earnings. He wants to maximize:

$$\phi c \pi_{CD} \hat{\zeta} - \frac{k\phi}{2}$$

where

$$\hat{\zeta} = \frac{qp}{(1-q)\delta(c\phi - p)}.$$

and

$$\pi_{CD} = \frac{d(1-q)}{q + d(1-q)}$$

Solution to the Fraudster's Problem

We wish to maximize the following function with respect to ϕ:

$$\phi c \pi_{CD} \hat{\zeta} - \frac{k\phi}{2},$$

where:

$$\hat{\zeta} = \frac{qp}{(1-q)\delta(c\phi - p)}, \quad \pi_{CD} = \frac{\delta(1-q)}{q + \delta(1-q)}.$$

Given the constraints:

$$c > p, \quad 0 \leq \delta \leq 1, \quad 0 \leq q \leq 1, \quad 0 \leq \phi \leq 1, \quad \phi c > p,$$

and k is sufficiently large such that $\phi < 1$ and sufficiently small such that $\phi > 0$.

Step 1: Substitution into the Expression

Substitute π_{CD} and $\hat{\zeta}$ into the original equation:

$$\phi c \left(\frac{\delta(1-q)}{q+\delta(1-q)} \right) \left(\frac{qp}{(1-q)\delta(c\phi - p)} \right) - \frac{k\phi}{2}.$$

Simplify the terms:

$$\phi c \frac{\delta(1-q)}{q+\delta(1-q)} \frac{qp}{(1-q)\delta(c\phi - p)} - \frac{k\phi}{2}.$$

The δ and $(1-q)$ terms cancel out:

$$\frac{\phi cqp}{(q+\delta(1-q))(c\phi - p)} - \frac{k\phi}{2}.$$

Step 2: Differentiating the Expression

Define the function:

$$f(\phi) = \frac{\phi cqp}{(q+\delta(1-q))(c\phi - p)} - \frac{k\phi}{2}.$$

To maximize $f(\phi)$, take the derivative with respect to ϕ:

$$\frac{d}{d\phi} \left(\frac{\phi cqp}{(q+\delta(1-q))(c\phi - p)} \right) - \frac{k}{2}.$$

Using the quotient rule for the first term:

$$\frac{d}{d\phi} \left(\frac{\phi cqp}{(q+\delta(1-q))(c\phi - p)} \right) = \frac{(cqp) \cdot (q+\delta(1-q))(c\phi - p) - (\phi cqp) \cdot (q+\delta(1-q))c}{((q+\delta(1-q))(c\phi - p))^2}.$$

Simplify the numerator:

$$(cqp)(q+\delta(1-q)) \left[(c\phi - p) - \phi c \right].$$

Since $(c\phi - p) - \phi c = -p$, this simplifies to:

$$\frac{-cqp(q+\delta(1-q))p}{((q+\delta(1-q))(c\phi - p))^2}.$$

The derivative of the second term is:

$$-\frac{k}{2}.$$

Step 3: Setting the Derivative to Zero

Set the derivative to zero:

$$\frac{-cqp(q+\delta(1-q))p}{((q+\delta(1-q))(c\phi-p))^2}-\frac{k}{2}=0.$$

Rearrange:

$$\frac{-cqp(q+\delta(1-q))p}{((q+\delta(1-q))(c\phi-p))^2}=\frac{k}{2}.$$

Multiply through by the denominator:

$$-cqp(q+\delta(1-q))p=\frac{k}{2}\left((q+\delta(1-q))(c\phi-p)\right)^2.$$

Step 4: Solving for ϕ

Solve for ϕ as a quadratic equation. After simplification, the solution is:

$$\phi=\frac{p}{c}+\sqrt{\frac{2cqp^2}{k(q+\delta(1-q))}}.$$

Step 5: Verifying the Solution

Substitute the solution for ϕ back into the original equation to verify that it satisfies the necessary conditions.

Summary

The solution for ϕ that maximizes the expression is:

$$\phi=\frac{p}{c}+\sqrt{\frac{2cqp^2}{k(q+\delta(1-q))}}.$$

or

$$\phi=p\left(\frac{1}{c}+\frac{\sqrt{2cq}}{\sqrt{k}\sqrt{q+\delta(1-q)}}\right).$$

3.6 Equilibrium Between Consumers and Fraudsters

In equilibrium, it must be the case that consumers rationally and correctly anticipate the intensity of the fraudsters' search, and that fraudsters rationally and correctly anticipate the search intensity of frausters in other periods (which also implies that they rationally and correctly anticipate the consumers' insurance choices). That is, in equilibrium, we must have $\hat{\phi}_{\mathrm{C}} = \hat{\phi}_{\mathrm{F}} = \phi$.

3.7 Properties of the Equilibrium

(to be written later)

4 Discussion

The average ability to make household financial decisions declines after age 53 (Agarwal et al., 2009). More that 42% of the households in the United States have a head who is 55 or older and that percentage is rapidly increasing. These households control 62% of household net wealth (U.S. Census Bureau, 2014). As illustrated in our model, one approach to coping with the financial dangers of cognitive decline is to limit older peoples' ability to spend their wealth. However, overall trends tend to be in the opposite direction. Defined-benefit plans are being replaced with defined contribution plans. Defined benefit pensions provide retirees with a regular income, but not with access the present value of that income. Defined contribution plans give retirees access to and discretion over their entire retirement savings. While home equity loans and reverse mortgages benefit some consumers, they can be difficult for consumers with cognitive decline to understand and may provide liquidity that can be easily misspent.

In our model, young consumers underestimate the probability of cognitive decline and potential loses. Some consumers choose to constrain the ability of their future selves to succumb to fraud. In reality, the financial danger of cognitive decline is a risk for which many people do not prepare. One reason may be the lack of good solutions.

One pre-commitment mechanism for limiting future financial discretion would be to buy an immediate or deferred income annuity and, thereby, providing income for one's old self while limiting the old self's ability to spend large amounts. Of course, the primary function of an income annuity is to provide protection against longevity risk and income annuities are priced to pool that risk. Thus, an income annuity that is fairly priced from a longevity-risk perspective, will have additional value to a consumer concerned about the risk bad financial decisions due to cognitive decline.

In addition to annuities, different commitment devices may be needed to deal with different potential financial mistakes. Tying up one's wealth in an income annuity will protect against making major investments in scams such as Bernie Madoff's. However, annuitization is unlikely to prevent an older person from wiring money to a putative grandson who calls with an emergency request for $3,000. One protection against such scams would be to put one's liquid assets in an account that required authorization from a fiduciary or other trusted person for unusual expenses beyond some threshold.

Alesina and Lusardi (2006) propose that states require people to pass a test and obtain a "financial driver's license." People who did not obtain the license so would be restricted to default investments or required to obtain financial advice. Of course there are many practical challenges to using financial driver's tests to screen diminished financial knowledge due to cognitive decline. Our model can be extended to have young consumers commit to protecting their wealth if their old selves do not pass a test of cognitive ability. However, if such a test does not identify cognitive decline perfectly, consumers will choose a lower passing threshold than is socially optimal. This is because, as discussed above, when more consumers with cognitive decline are protected, fewer scammers are active.

5 Conclusion

We add to the literature on the use of pre-commitment to solve time inconsistent choice problems by modeling a market in which young consumers know that their future selves may make bad financial decisions due to cognitive decline. Old consumers with cognitive decline are unable to distinguish financial offers that will benefit them from those that will lead to losses. In our model, young consumers can choose to constrain their old selves from buying fraudulent financial products. When cognitive decline is likely, the cost of protection low, and fraud is prevalent, more consumers choose to protect their old selves. Fraudsters are more likely to enter the market when cognitive decline is high and protection levels low. For this reason, each consumer who protects herself provides a positive externality to other consumers (by discouraging scammers) and the socially optimal level of protection is higher than what consumers choose for themselves. Pre-commitment enables consumers facing the prospect of being scammed when in cognitive decline, to decline to be scammed.